1. $Q = 9.28 \times 10^{6}$

$R = 1.9 \times 10^{5}$

<u>Soln</u>

$Q = 92.8 \times 10^{5}$

$R = 1.9 \times 10^{5}$

a) $Q + R = (92.8 + 1.9) \times 10^{5} = 94.7 \times 10^{5} = 9.47 \times 10^{6}$ J

(b) $Q - R = (92.8 - 1.9) \times 10^{5} = 90.9 \times 10^{5} = 9.09 \times 10^{6}$ J

c) $QR = (92.8 \times 1.9) \times 10^{5+5} = 176.32 \times 10^{10} = 1.7632 \times 10^{12}$

(d) $Q/R = \left(\frac{92.8}{1.9}\right) \times \frac{\cancel{10^5}}{\cancel{10^5}} = 48.84$

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(a)

② No. of grains placed on nth square $x = 2^{n-1}$ --- ①

for first square $x = 2^{1-1} = 1$

for second square $x = 2^{2-1} = 2$ and so on

Since chessboard has 64 squares (and it is the last square to fill), grains filled in last (64th square is given by:

$$x = 2^{64-1} = 2^{63} = 9.22 \times 10^{18} \text{ grains of rice.}$$

(b) Total number of grains filled in whole chessboard

$$S = (1) + (2) + (2\times2) + (2\times2\times2) + (2\times2\times2\times2) + \dots 2^{n-1} \text{ --- ②}$$

Taking the double of this sum (multiply each term by 2)

$$2S = (2\times1) + (2\times2) + (2\times2\times2) + (2\times2\times2\times2) + \dots + 2^{n} \text{ --- ③}$$

Eqn ③ - ②

$$2S - S = [(2\times1) + (2\times2) + (2\times2\times2) + (2\times2\times2\times2)] - (1) + (2) + (2\times2) + (2\times2\times2) + (2\times2\times2\times2)$$

$$S = 2^{n} - 1$$

Since $n = 64$

$$S = 2^{64} - 1 = 1.84 \times 10^{19} \text{ grains of rice}$$

(c) Given 1kg of rice has 15000 grains

1kg = 1.24 litre

mass of 1 grain $= \frac{1}{15000}$ kg

volume of 1 grain of rice $\frac{1}{15000} \times 1.24 = 8.26 \times 10^{-5}$ litres

volume of all grains $= 8.26 \times 10^{-5} \times 1.84 \times 10^{19}$

$$= 1.519 \times 10^{15} = 1.52 \times 10^{19}\ m^3$$

Consider the surface of the earth is surrounded by this volume of grains

After symmetrical covering by grains all over earth, new radius $= R_{rice}$

Δ volume of earth $= V_{rice} - V_{earth}$

$= \frac{4}{3}\pi R_{rice}^3 - \frac{4}{3}\pi R^3$

$\frac{3}{4\pi} \times 1.52\times10^{15} = \cancel{\frac{4}{3}\pi}\left(R_{rice}^3 - R^3\right]$ $\quad R = 6371\ km = 6371000$

$3.63\times10^{14} = R_{rice}^3 - (6371000)^3$

$2.585969\times10^{20} = R^3_{rice}$.

$6371000.003m = R_{rice} = 6371000.003\ m$

$d = R_{rice} - R_{earth}$

$= 6371000.003 - 6371000 = 0.003\ m = 3mm$

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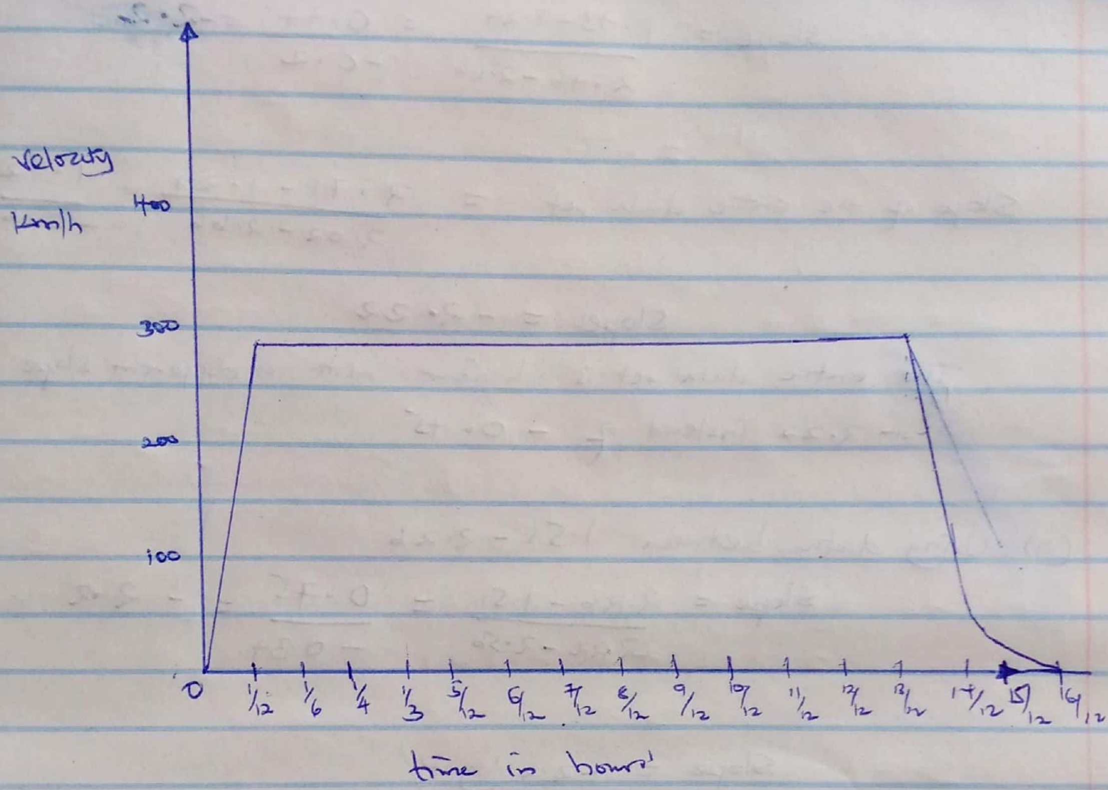
3. Distance between New York and Washington by air = 213 miles / 343 km / 185 nautical miles

The flight time is 1hr 10mins = $\frac{7}{6}$ hours



$$\text{Avg speed} = \frac{343}{7/6} = 294 \text{ km/h}$$

Sharp corners mean the plane attained a maximum speed and travelled by that speed for the remaining time and the last corner means the plane's speed began dropping from maximum speed so as to attain a good landing velocity. The passengers have to be properly on belts to avoid shifting from seats.

b) $d = 24.5 + \left(\frac{t-5}{60} \times 294\right) = 24.5 + 4.9t - 24.5$

$\Rightarrow d = 4.9t$

c) $d = 343 - 4.9t$, because we subtract the covered distance from the total to get remaining distance.

for $25 \leq d \leq 75$

a) $\frac{\Delta T}{\Delta d} = \frac{5.1 - 5.5}{75 - 25} = \frac{-0.4}{50} = -0.008°c/m$

for $100 \leq d \leq 200$

$\Delta T/\Delta d = \frac{6.0 - 5.1}{200 - 100} = \frac{0.9}{100} = 0.009°c/m$

Between a ~~temperature~~ depth of 25-75, temperatures decreases and increases again from 100m downwards. This drop could have been due to erroneous Measurement.

(b) $T = 5.5 - (0.005 \times (D - 25))$

c) Applying the above formula

$T = 5.5 - 0.005 \times (300 - 25)$

$= 4.125°c$

The engineer is out by $7.00 - 4.125 = 2.875°c$

(d) From 150 - 300 rate of increase of temperature is Constant

$\frac{\Delta t}{\Delta d} = \frac{7.00 - 5.5}{300 - 150} = \frac{1.5}{150} = 0.01$

$T = 5.5 + 0.01 (D - 150)$ where T is temperature at a given distance from 150 m

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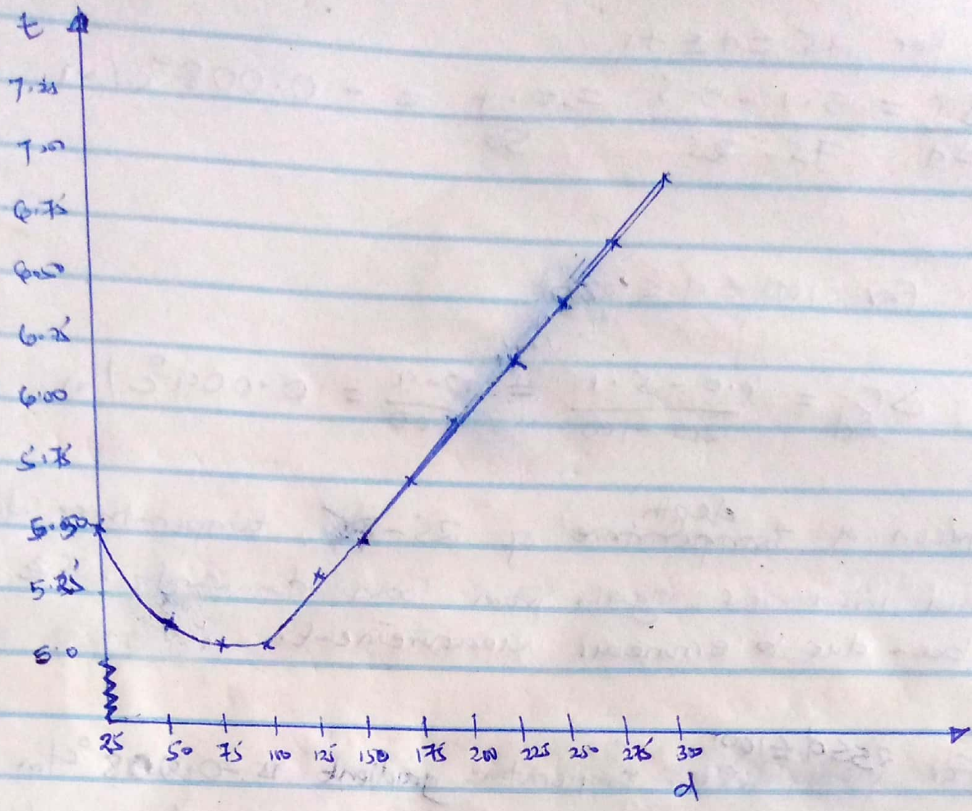

(5) 28 miles/gallon – 60's

9.2 litres/100km – modern

1 mile = 1.6093 km and 1 gallon = 3.785 litre

60s car = 28 miles/gallon = $\frac{28 \times 1.6093}{3.785}$ km/litre = 11.90 km/litre

Modern car consumption = $\frac{100\text{ km}}{9.2\text{ l}}$ = 10.869 km/litre

The 60s car had a better fuel consumption

(4) (a) Weight B is closer to the ceiling

(b) Weight A makes largest oscillation

(c) Weight A makes fastest oscillation

(d) Weight A : $d(t) = 20 \sin(4\pi t)$

(6) for (a) height of A from ceiling = 20 cm

height of B from ceiling = 10 cm

so B is closer

(b) largest oscillation = Max value - Min value

for weight A, oscillation $= 30-10 = 20cm$
for weight B, oscillation $= 15-5 = 10cm$.
So weight A makes largest oscillation.

c) In 2 sec weight A makes 4 complete oscillation
In 2 sec weight B makes 1 complete oscillation,
So weight A makes fastest oscillation.

d) Sinusoidal function
$d(t) = A \sin(\omega t)$ because phase is 0, as a function starts from $x=0$ line (y-axis)

For weight A, A = mean value of 20
Since time required for 4 complete oscillations $= 2s$
T, time required for 1 oscillation $= 0.5s$
$\omega = 2\pi f = 2\pi/t = 2\pi \times 1/0.5 = 4\pi$
for A : $d(t) = 20 \sin(4\pi t)$
for weight B, A =10
Time required for one oscillation $= 2s$
$\omega = 2\pi/T = 2\pi/2 = \pi$

for B $d(t) = 10 \sin(\pi t)$

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7.

Log P	1.29	1.51	1.73	2.26	2.71
log Q	2.66	2.56	2.46	2.22	2.02

Between 1.29 − 1.73

$$\text{Slope} = \frac{1.73-1.29}{2.46-2.66} = \frac{0.44}{-0.2} = -2.2$$

$$\text{Slope of the entire data set} = \frac{2.71-1.29}{2.02-2.66} = \frac{1.42}{-0.64}$$

$$\text{Slope} = -2.22$$

The entire data set is Linear with a different slope of −2.22 instead of −0.45

(b) Using data between 1.51 − 2.26

$$\text{Slope} = \frac{2.26-1.51}{2.22-2.56} = \frac{0.75}{-0.34} = -2.2$$

$$\text{Slope} = -2.2 \log P / \log Q$$

$$\log Q = -2.2 \log P$$

(c) $\log Q = -2.2 \log P$

$$Q = P^{-2.2} \Rightarrow Q = \frac{1}{P^{2.2}}$$

(d) $P^{2.2} = 1/Q \Rightarrow P = \sqrt[2.2]{1/Q} = \left[\frac{1}{Q}\right]^{\frac{1}{2.2}} = Q^{2.2}$

(e) P is non linear

(f) The data set can be multiplied by a conversion factor of 2.303 to convert from log to base 10 to log to base e.

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8. Carbon -~~14~~ remaining after X-years

We use the points (0,100) and (5730,50)

$$y = A_0 e^{kx}$$

$$50 = 100 e^{5730k}$$

$$0.5 = e^{5730k} \Rightarrow 5730k = \ln 0.5$$

$$k = \frac{\ln 0.5}{5730} = -0.0001209$$

$$\text{So } y = 100 e^{-0.000120968} \quad \text{--- (i)}$$

Vermeer died in 1675

It means $2021 - 1675 = 346$

substitute $x = 346$ in --(i) we get

$$y = 100 e^{-0.000120698 \times 346} = 95.91\%$$

But it is given that the painting contains 98.5% of Carbon-14, so the painting is fake.

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